## Cambridge IGCSE ${ }^{\text {TM }}$

CANDIDATE
NAME
CENTRE NUMBER

|  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- |

CANDIDATE NUMBER

## MATHEMATICS

0580/04
Paper 4 (Extended)
SPECIMEN PAPER
For examination from 2020
2 hours 30 minutes

You must answer on the question paper.
You will need:
Geometrical instruments

## INSTRUCTIONS

- Answer all questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do not use an erasable pen or correction fluid.
- Do not write on any bar codes.
- You should use a calculator where appropriate.
- You may use tracing paper.
- You must show all necessary working clearly.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- $\quad$ For $\pi$, use either your calculator value or 3.142.


## INFORMATION

- The total mark for this paper is 130 .
- The number of marks for each question or part question is shown in brackets [ ].

This document has 20 pages. Blank pages are indicated.

1 (a) Kristian and Stephanie share some money in the ratio 3:2. Kristian receives \$72.
(i) Work out how much Stephanie receives.
\$.
(ii) Kristian spends $45 \%$ of his $\$ 72$ on a computer game.

Calculate the price of the computer game.
\$.
(iii) Kristian also buys a meal for $\$ 8.40$.

Calculate the fraction of the $\$ 72$ Kristian has left after buying the computer game and the meal.
Give your answer in its lowest terms.
(iv) Stephanie buys a book in a sale for $\$ 19.20$.

This sale price is after a reduction of $20 \%$.

Calculate the original price of the book.
(b) Boris invests $\$ 550$ at a rate of $2 \%$ per year simple interest.

Calculate the value of the investment at the end of 10 years.
\$
(c) Marlene invests $\$ 550$ at a rate of $1.9 \%$ per year compound interest.

Calculate the value of the investment at the end of 10 years.
\$
[2]
(d) Hans invests $\$ 550$ at a rate of $x \%$ per year compound interest.

At the end of 10 years, the value of the investment is $\$ 638.30$, correct to the nearest cent.
Find the value of $x$.
$x=$

2 (a) 200 students estimate the volume, $V \mathrm{~m}^{3}$, of a classroom. The cumulative frequency diagram shows their results.


Use the graph to find an estimate of
(i) the median,
$\qquad$
(ii) the interquartile range,
$\qquad$
(iii) the 60th percentile,
$\qquad$
(iv) the number of students who estimate that the volume is greater than $300 \mathrm{~m}^{3}$.
(b) The 200 students also estimate the total area, $A \mathrm{~m}^{2}$, of the windows in the classroom. The table shows their results.

| Area $\left(A \mathrm{~m}^{2}\right)$ | $20<A \leqslant 60$ | $60<A \leqslant 100$ | $100<A \leqslant 150$ | $150<A \leqslant 250$ |
| :--- | :---: | :---: | :---: | :---: |
| Frequency | 32 | 64 | 80 | 24 |

(i) Calculate an estimate of the mean.

You must show all your working.
$\qquad$
(ii) Complete the histogram to show the information in the table.

(iii) Two students are chosen at random from those students that estimated the area of the windows to be more than $100 \mathrm{~m}^{2}$.

Find the probability that one of the two students estimates the area to be greater than $150 \mathrm{~m}^{2}$ and the other student estimates the area to be $150 \mathrm{~m}^{2}$ or less.
$3 \mathrm{f}(x)=\frac{20}{x}+x, x \neq 0$
(a) Complete the table.

| $x$ | -10 | -8 | -5 | -2 | -1.6 | 1.6 | 2 | 5 | 8 | 10 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{f}(x)$ | -12 | -10.5 | -9 | -12 | -14.1 | 14.1 | 12 |  |  | 12 |

(b) On the grid, draw the graph of $y=\mathrm{f}(x)$ for $-10 \leqslant x \leqslant-1.6$ and $1.6 \leqslant x \leqslant 10$.

(c) Using your graph, solve the equation $\mathrm{f}(x)=11$.

$$
x=. . . . . . . . . . . . . . . . . ~ o r ~ x=
$$

(d) $k$ is a prime number and $\mathrm{f}(x)=k$ has no solutions.

Find the possible values of $k$.
(e) The gradient of the graph of $y=\mathrm{f}(x)$ at the point $(2,12)$ is -4 .

Write down the coordinates of the other point on the graph of $y=\mathrm{f}(x)$ where the gradient is -4 .
$\qquad$
(f) (i) The equation $\mathrm{f}(x)=x^{2}$ can be written as $x^{3}+p x^{2}+q=0$.

Show that $p=-1$ and $q=-20$.
(ii) On the grid opposite, draw the graph of $y=x^{2}$ for $-4 \leqslant x \leqslant 4$.
(iii) Using your graphs, solve the equation $x^{3}-x^{2}-20=0$.

$$
x=
$$

(iv)


NOT TO
SCALE

The diagram shows a sketch of the graph of $y=x^{3}-x^{2}-20$.
$P$ is the point $(n, 0)$.
Write down the value of $n$.

$$
\begin{equation*}
n= \tag{1}
\end{equation*}
$$


(a) (i) Draw the reflection of triangle $T$ in the line $x=0$.
(ii) Draw the rotation of triangle $T$ about $(-2,-1)$ through $90^{\circ}$ clockwise.
(b) Describe fully the single transformation that maps triangle $T$ onto triangle $U$.
$\qquad$
$\qquad$

5 (a)


The perimeter of the rectangle is 80 cm .
The area of the rectangle is $A \mathrm{~cm}^{2}$.
(i) Show that $x^{2}-40 x+A=0$.
(ii) When $A=300$, solve the equation $x^{2}-40 x+A=0$ by factorising.

$$
\begin{equation*}
x= \tag{3}
\end{equation*}
$$

$\qquad$ or $x=$
(iii) When $A=200$, solve the equation $x^{2}-40 x+A=0$ using the quadratic formula. Show all your working and give your answers correct to 2 decimal places.

$$
x=
$$

$\qquad$ or $x=$
(b) A car completes a 200 km journey at an average speed of $x \mathrm{~km} / \mathrm{h}$.

The car completes the return journey of 200 km at an average speed of $(x+10) \mathrm{km} / \mathrm{h}$.
(i) Show that the difference between the time taken for each of the two journeys is $\frac{2000}{x(x+10)}$ hours.
(ii) Find the difference between the time taken for each of the two journeys when $x=80$. Give your answer in minutes and seconds.
$\min$

$O P Q R$ is a rectangle and $O$ is the origin.
$M$ is the midpoint of $R Q$ and $P T: T Q=2: 1$.
$\overrightarrow{O P}=\mathbf{p}$ and $\overrightarrow{O R}=\mathbf{r}$.
(a) Find, in terms of $\mathbf{p}$ and/or $\mathbf{r}$, in its simplest form
(i) $\overrightarrow{M Q}$,

$$
\begin{equation*}
\overrightarrow{M Q}= \tag{1}
\end{equation*}
$$

(ii) $\overrightarrow{M T}$,

$$
\begin{equation*}
\overrightarrow{M T}= \tag{1}
\end{equation*}
$$

(iii) $\overrightarrow{O T}$.

$$
\overrightarrow{O T}=
$$

(b) $R Q$ and $O T$ are extended and meet at $U$.

Find the position vector of $U$ in terms of $\mathbf{p}$ and $\mathbf{r}$.
Give your answer in its simplest form.
(c) $\overrightarrow{M T}=\binom{2 k}{-k}$ and $|\overrightarrow{M T}|=\sqrt{180}$.

Find the positive value of $k$.

$$
\begin{equation*}
k= \tag{3}
\end{equation*}
$$

$$
\mathrm{f}(x)=2 x+1 \quad \mathrm{~g}(x)=x^{2}+4 \quad \mathrm{~h}(x)=2^{x}
$$

(a) Solve the equation $\mathrm{f}(x)=\mathrm{g}(1)$.

$$
x=
$$

(b) Find $\mathrm{f}^{-1}(x)$.

$$
\mathrm{f}^{-1}(x)=
$$

(c) Find $\operatorname{gf}(x)$ in its simplest form.
(d) Solve the equation $\mathrm{h}^{-1}(x)=0.5$.

$$
\begin{equation*}
x= \tag{1}
\end{equation*}
$$

(e) $\frac{1}{\mathrm{~h}(x)}=2^{k x}$

Write down the value of $k$.

$$
\begin{equation*}
k= \tag{1}
\end{equation*}
$$

8 The grid shows the graph of $y=\cos x$ for $0^{\circ} \leqslant x \leqslant 360^{\circ}$.

(a) Solve the equation $3 \cos x=1$ for $0^{\circ} \leqslant x \leqslant 360^{\circ}$. Give your answers correct to 1 decimal place.
and
(b) On the same grid, sketch the graph of $y=\sin x$ for $0^{\circ} \leqslant x \leqslant 360^{\circ}$.

9


The diagram shows a trapezium $A B C D$.
$A B$ is parallel to $D C$.
$A B=55 \mathrm{~m}, B D=70 \mathrm{~m}$, angle $A B D=40^{\circ}$ and angle $B C D=32^{\circ}$.
(a) Calculate $A D$.

$$
A D=
$$

m [4]
(b) Calculate $B C$.
$B C=$
m [4]
(c) Calculate the area of $A B C D$.
$m^{2}[3]$
(d) Calculate the shortest distance from $A$ to $B D$.
m [2]

10 (a) Show that the volume of a metal sphere of radius 15 cm is $14140 \mathrm{~cm}^{3}$, correct to 4 significant figures.
[The volume, $V$, of a sphere with radius $r$ is $V=\frac{4}{3} \pi r^{3}$.]
(b) (i) The sphere is placed inside an empty cylindrical tank of radius 25 cm and height 60 cm . The tank is filled with water.


NOT TO
SCALE

Calculate the volume of water needed to fill the tank.
$\mathrm{cm}^{3}$ [3]
(ii) The sphere is removed from the tank.


NOT TO
SCALE

Calculate the depth, $d$, of water in the tank.

$$
d=
$$

$\qquad$ cm [2]
(c) The diagram below shows a solid circular cone and a solid sphere.


NOT TO
SCALE

The cone has radius $5 x \mathrm{~cm}$ and height $12 x \mathrm{~cm}$.
The sphere has radius $r \mathrm{~cm}$.
The cone has the same total surface area as the sphere.
Show that $r^{2}=\frac{45}{2} x^{2}$.
[The curved surface area, $A$, of a cone with radius $r$ and slant height $l$ is $A=\pi r l$.] [The surface area, $A$, of a sphere with radius $r$ is $A=4 \pi r^{2}$.]

11 A curve has equation $y=x^{3}-6 x^{2}+16$.
(a) Find the coordinates of the two turning points.
(........... , ...........) and (............ ,...........) [
(b) Determine whether each of the turning points is a maximum or a minimum. Give reasons for your answers.

